

Three Wheeled Vehicle Dynamics: Static Stability Factor Improvement

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Abstract: Three Wheeled Vehicles are gaining popularity across various Asian and African countries. They are characterized for their relatively low susceptibility to rollover when compared to Four Wheeled Vehicles. Establishing an analytical criterion based on architectural parameters such as wheelbase, rear track, masses, and center of gravity coordinates is crucial. This criterion assists manufacturers, particularly during the initial design stages, providing valuable guidance for optimizing vehicle stability. The Static Stability Factor serves as a prominent design criterion in the automotive sector, offering an assessment of a vehicle's resistance to rollover incidents. This paper aims to enhance the existing criterion initially proposed by Huston et al., particularly by addressing scenarios where a load is unevenly distributed on the vehicle's tray. The findings of this study underscore the high sensitivity of such vehicles to unsymmetrical loading configurations.

Keywords: Three Wheeled Vehicle; Rollover; Active Safety; Static Stability Factor.

1- Introduction

In 2023, the global Three Wheeled Vehicle (TWV) market achieved a substantial size of US\$ 10.7 Billion [1]. Projecting ahead, IMARC Group anticipates significant growth, with the market expected to reach US\$ 22.5 Billion by 2032. This forecast suggests a robust Compound Annual Growth Rate (CAGR) of 8.6% during the period from 2024 to 2032. Several key factors are driving this expansion, including an increasing emphasis on fuel efficiency, rising demand for economical mobility solutions, enhanced versatility of TWV, the emergence of eco-friendly alternatives, a growing need for efficient transportation modes, and supportive governmental policies [2, 3].

However, despite the numerous advantages offered by TWV, considerable attention must be devoted to addressing safety concerns. Unlike their Four Wheeled counterparts, TWV are inherently more prone to accidents [4, 5]. The heightened risk of rollover, particularly noticeable during maneuvers, presents a substantial safety challenge. Hence, there exists a critical need for continuous refinement of TWV safety measures, including ongoing enhancements in design and the integration of advanced features aimed at enhancing stability and reducing potential risks.

The Static Stability Factor (SSF) is a crucial metric in vehicle dynamics [6], particularly in assessing a vehicle's resistance to rollover during cornering or maneuvers. It represents the maximum lateral acceleration (expressed in g-force unit) that the vehicle can withstand. The Rollover threshold (SSF) varies from 0.4 g for heavy trucks to 1.7 g for sports car [7]. A higher SSF indicates greater resistance to rollover [8], enhancing the vehicle's stability and safety. Manufacturers and engineers utilize this metric extensively in designing and evaluating vehicles, aiming to optimize stability while ensuring optimal handling and performance on various road conditions.

The SSF of a Four Wheeled Vehicle [6] is calculated using the formula :

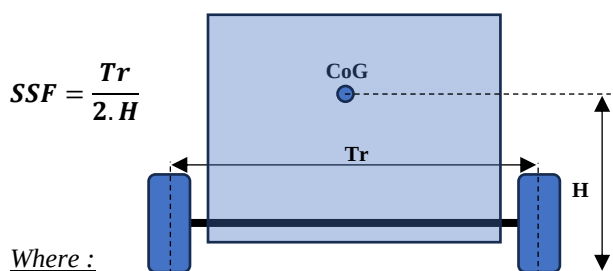


Figure 1. Static Stability Factor Diagram – Rear View

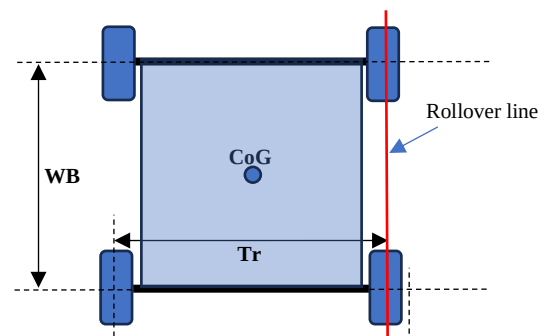


Figure 2. Static Stability Factor Diagram – Top View

Where :

Tr : Track width,

H : Height of the center of gravity of the vehicle to the ground,

WB : Wheelbase.

For Four Wheeled Vehicles, engineers can determine the SSF by considering just two key parameters: the rear track width and the height of the center of gravity. This simplified process, offers engineers a notable advantage, enabling them to evaluate rollover risks in the early design phases, even before defining the specifications of suspension [9] and tire characteristics that are complicated to define in the early stages of vehicle design.

There are two categories of TWV [10]: the Delta variant, characterized by two rear wheels and one front wheel, and the Tadpole variant, distinguished by two front wheels and one rear wheel.



Figure 3. Basic types of the TWV: (a) Delta variant and (b) Tadpole variant

The Delta variant provides superior versatility and maneuverability, attributed to its front wheel's ability to rotate at larger angles, unlike the Tadpole variant. However, its drawback becomes apparent in its diminished stability during cornering or maneuvers. The Delta variant of TWV stands out as the predominant choice for public and commercial transport in emerging countries.



Figure 4. Examples of type urban TWV in Nepal, Tanzania and Liberia [2]

In their 1982 paper [11], Huston et al. showcased that the SSF of a Delta variant TWV can be determined using the formula :

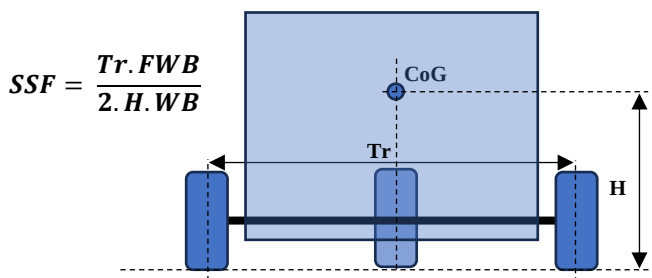


Figure 5. Static Stability Factor Diagram - TWV - Rear view

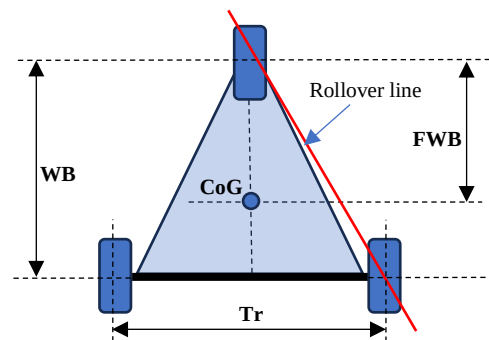


Figure 6. Static Stability Factor Diagram - TWV - Top view

Where :

Tr : Track width,

H : Height of the center of gravity of the vehicle to the ground,

WB : Wheelbase,

FWB : Front axle to CoG distance.

In addition to track width and center of gravity height, TWV rely on the longitudinal position of the center of gravity for lateral stability [12]. Shifting the center of gravity rearward notably improves this stability, reducing the risk of rollover during maneuvers. This emphasizes the crucial role of weight distribution in enhancing overall vehicle performance and safety. Paul G. Van Valkenburgh et al was undertaken a study to assess the rollover thresholds of multiple three-wheeled

vehicles [13]. However, it appears that the obtained values are overly optimistic. This discrepancy arises from the vehicles' notably low center of gravity height and wider rear track in comparison to standard TWV.

The design of TWV is also influenced by the lateral position of the center of gravity, this is what Martín R. Licea demonstrated with a different approach [14].

The objective of this study is to enhance the SSF to incorporate this lateral aspect. Subsequently, a comparative analysis is conducted between the modified criterion and the original one to illustrate the discrepancies between the two criteria.

2- Materials and Methods

S.J. Ojolo et al. conducted a studies [15, 16] to illustrate the SSF criterion as initially proposed by Huston et al., employing both a static and a dynamic approach. Additionally, their research introduced a test procedure for determining the rollover threshold.

In this section, we will summarize the findings of S.J. Ojolo et al. and propose adjustments to account for the lateral component of the center of gravity.

a. Mathematical demonstration of the SSF criterion

A Free Body Diagram (FBD) is a crucial instrument in physics and engineering, used to analyze the forces affecting an object. It simplifies the process by isolating the object and illustrating all exerted forces as vector arrows. This visual representation aids in determining the resultant force on the object, thus enabling predictions about its motion. Mastery in constructing and interpreting FBD is vital for solving problems concerning equilibrium, motion, or dynamics in the realms of mechanics and physics.

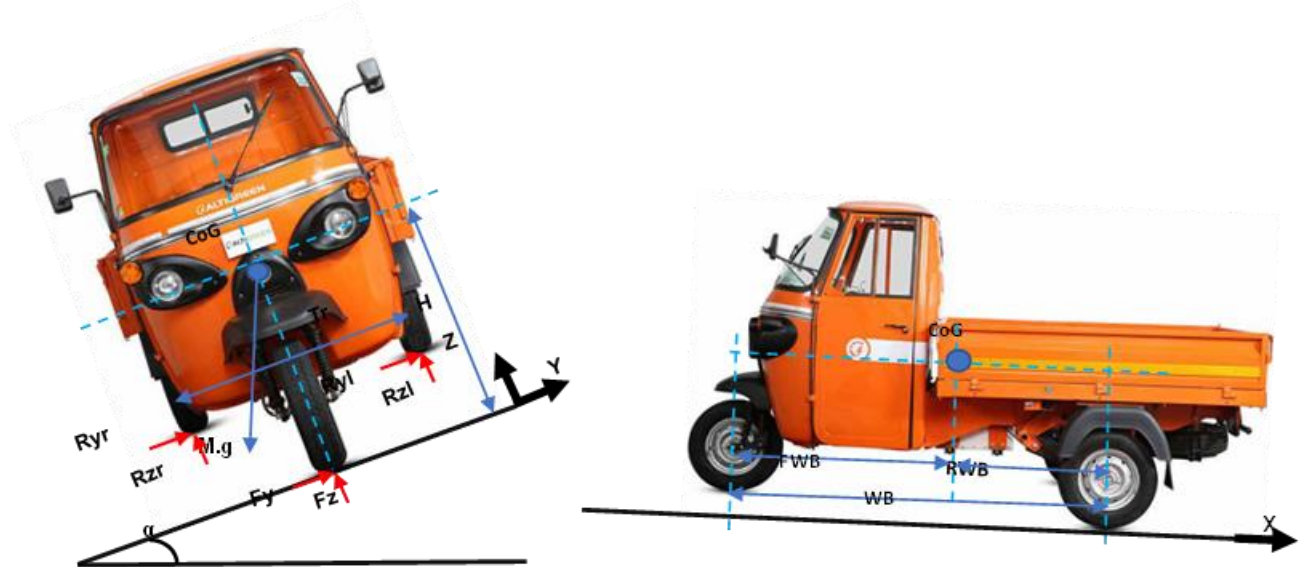


Figure 7. TWV on a banked road

Figure 7 depict a TWV [17] on a banked road. The aim is to create a graphical representation showcasing the various forces influencing the vehicle to establish the stability criterion.

Table 1. Nomenclature

Symbol	Description
CoG	Center of Gravity
M.g	Weight
H	Center of Gravity height to the ground
Tr	Rear Track
WB	Wheelbase
FWB	Front axle to CoG distance
RWB	Rear axle to CoG distance
Fy	Front lateral force
Fz	Front vertical force
Ryr	Rear right lateral force
Rzr	Rear right vertical force
Ryl	Rear left lateral force
Rzl	Rear left vertical force
α	Angle of banking

By applying Newton's first law on both the horizontal axis (Y) and the vertical axis (Z) allows us to formulate :

$$F_y + R_{yr} + R_{yl} = M \cdot g \cdot \sin \alpha \quad (1)$$

$$F_z + R_{zr} + R_{zl} = M \cdot g \cdot \cos \alpha \quad (2)$$

Calculating front reactions:

$$F_y = \frac{RWB}{WB} \cdot M \cdot g \cdot \sin \alpha \quad (3)$$

$$F_z = \frac{RWB}{WB} \cdot M \cdot g \cdot \cos \alpha \quad (4)$$

Calculating rear reactions:

$$R_{yr} + R_{yl} = \frac{FWB}{WB} \cdot M \cdot g \cdot \sin \alpha \quad (5)$$

$$R_{zr} + R_{zl} = \frac{FWB}{WB} \cdot M \cdot g \cdot \cos \alpha \quad (6)$$

By applying moments around the X axis, it is possible to formulate :

$$H \cdot (F_y + R_{yr} + R_{yl}) = \frac{Tr}{2} \cdot (R_{zr} - R_{zl}) \quad (7)$$

By injecting equations (1) and (6) into equation (7) to eliminate "Rzl ", we can write :

$$H \cdot (M \cdot g \cdot \sin \alpha) = \frac{Tr}{2} \cdot (R_{zr} - \left(\frac{FWB}{WB} \cdot M \cdot g \cdot \cos \alpha - R_{zr} \right)) \quad (8)$$

By simplifying:

$$R_{zr} = \frac{FWB}{2 \cdot WB} \cdot (M \cdot g \cdot \cos \alpha) + \frac{H}{Tr} \cdot (M \cdot g \cdot \sin \alpha) \quad (9)$$

By injecting equations (1) and (6) into equation (7) to eliminate "Rzr ", we can write :

$$H \cdot (M \cdot g \cdot \sin \alpha) = \frac{Tr}{2} \cdot \left(\left(\frac{FWB}{WB} \cdot M \cdot g \cdot \cos \alpha - R_{zl} \right) - R_{zl} \right) \quad (10)$$

By simplifying:

$$R_{zl} = \frac{FWB}{2 \cdot WB} \cdot (M \cdot g \cdot \cos \alpha) - \frac{H}{Tr} \cdot (M \cdot g \cdot \sin \alpha) \quad (11)$$

Rollover threshold is reached when the vertical force of one of the left or right wheels reaches zero :

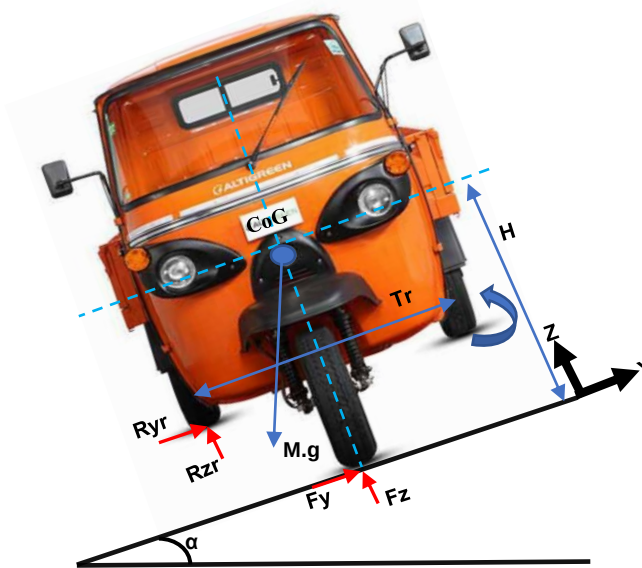


Figure 8. TWV at limit of rollover

If the vehicle does not slide as the banking angle " α " is increases due to the the tire's lateral grip margin [18], at limit of rollover " $Rz_l = 0$ ", therefore equation (11) becomes:

$$\frac{FWB}{2.WB} \cdot (M \cdot g \cdot \cos \alpha) = \frac{H}{Tr} \cdot (M \cdot g \cdot \sin \alpha) \quad (12)$$

By simplifying:

$$\frac{FWB}{2.WB} \cdot \cos \alpha = \frac{H}{Tr} \cdot \sin \alpha \quad (13)$$

Therefore, the Static Stability Factor (SSF) for the three wheeled vehicle is defined as :

$$SSF = \frac{Tr.FWB}{2.H.WB}$$

In addition to track width and center of gravity height, through this mathematical analysis, it has been established that the stability of TWV is directly influenced by the distribution of mass between the front and rear axles.

b. Improvement of the criterion

The idealization of the model by assuming a symmetrical distribution of the weight relative to the longitudinal axis of the vehicle may lead to an accurate estimation of the SSF. The purpose of this section is to propose a model improvement by considering the lateral coordinates of the center of gravity.

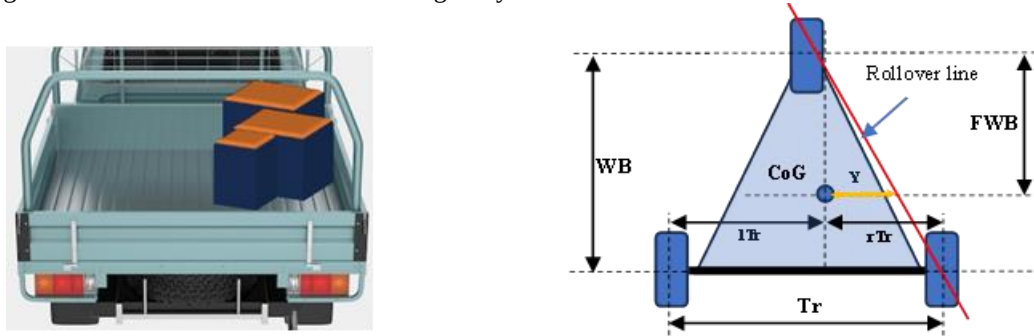


Figure 9. TWV with unsymmetrical load distribution

The unsymmetrical distribution of the load within the vehicle's tray results in a lateral displacement of the center of gravity. Consequently, the vehicle's stability will decrease as the lateral distance between the center of gravity and the rollover line (distance Y in Figure 9) diminishes on the loaded side. In other words, the vehicle's stability will vary depending on whether you make a left or right turn.

Table 2. Additional nomenclature

Symbol	Description
rTr	CoG to rear right wheel distance
lTr	CoG to rear left wheel distance

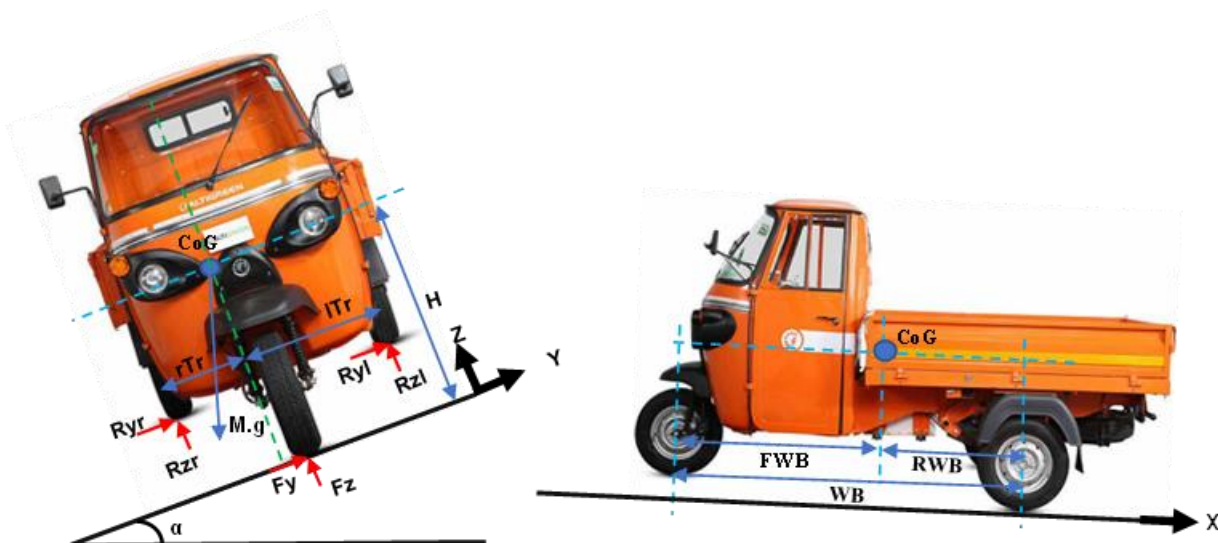


Figure 10. TWV with unsymmetrical load distribution about X-Axis

By applying moments around the X axis, it is possible to formulate :

$$H.(F_y + R_{yr} + R_{yl}) = R_{zr}.rTr - R_{zl}.lTr - F_z.\frac{lTr-rTr}{2} \quad (14)$$

Unlike equation (7), the front vertical reaction "Fz" creates a moment due to the lateral displacement of the center of gravity: $F_z.\frac{lTr-rTr}{2}$

By replacing the terms, one can write :

$$H.m.g.\sin\alpha = (\frac{FWB}{WB}.M.g.\cos\alpha - R_{zl}).rTr - R_{zl}.lTr - \frac{RWB}{WB}.M.g.\cos\alpha.\frac{lTr-rTr}{2} \quad (15)$$

By developing:

$$H.m.g.\sin\alpha = -R_{zl}.rTr - R_{zl}.lTr + (\frac{FWB}{WB}.M.g.\cos\alpha).rTr - \frac{RWB}{WB}.M.g.\cos\alpha.\frac{lTr-rTr}{2} \quad (16)$$

By simplifying:

$$H.m.g.\sin\alpha = -R_{zl}.Tr + M.g.\cos\alpha.(rTr.\frac{FWB}{WB} - \frac{lTr-rTr}{2}.\frac{RWB}{WB}) \quad (17)$$

We can rewrite the last term as:

$$rTr.\frac{FWB}{WB} - \frac{lTr-rTr}{2}.\frac{RWB}{WB} = rTr - \frac{Tr.RWB}{2.WB} \quad (18)$$

By injecting equation (18) into equation (17) and simplifying:

$$R_{zl} = \frac{M.g.\cos\alpha}{Tr}.\left(rTr - \frac{Tr.RWB}{2.WB}\right) - \frac{H}{Tr}.m.g.\sin\alpha \quad (19)$$

At limit of rollover "Rzl = 0", therefore equation (19) becomes after simplifying:

$$SSF_{LT} = \frac{rTr}{H} - \frac{RWB.Tr}{2.WB.H}$$

This is the SSF for the Delta variant TWV, incorporating the lateral coordinates of the center of gravity during a left turn.

By employing the same mathematical approach, it becomes evident that the SSF for the Delta variant TWV negotiating a right turn can be formulated as follows:

$$SSF_{RT} = \frac{lTr}{H} - \frac{RWB.Tr}{2.WB.H}$$

To establish a formulation applicable in a general context, we might express it as follows:

$$SSF = \frac{\min\{rTr; lTr\}}{H} - \frac{RWB.Tr}{2.WB.H}$$

Where :

rTr : CoG to rear right wheel distance,

lTr : CoG to rear left wheel distance,

H : Height of the center of gravity of the vehicle to the ground,

RWB : Rear axle to CoG distance,

Tr : Track width,

WB : Wheelbase.

3- Results and Discussion

Within this section, our primary goal is to conduct a comparison between the standard SSF and its upgraded version, with a specific emphasis on the heightened sensitivity of TWV to unsymmetrical loading. To achieve this aim, we will utilize the findings from the study conducted by A. Ekuase et al. [7], where they developed a TWV and meticulously documented its architectural parameters for our analysis.

Figure 11 provides a visual representation of this vehicle, showcasing its various architectural parameters, including wheelbase, rear track, and mass distribution. These parameters (table 3) will serve as the basis for our analytical analysis.

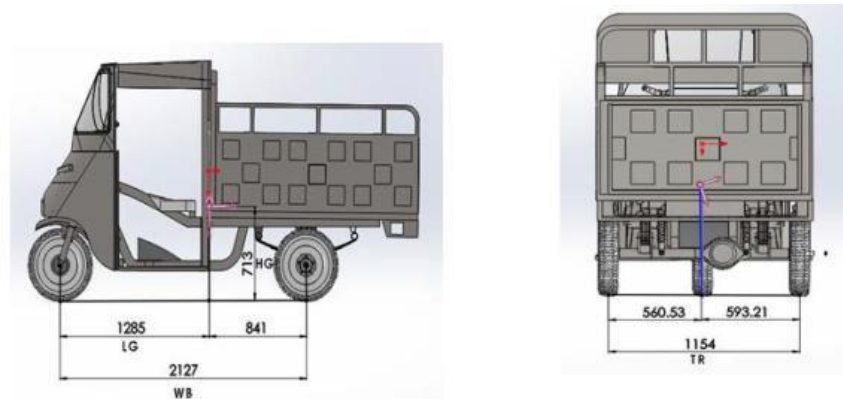


Figure 11. Architecture parameters of the TWV

Table 3. Architecture parameters of the TWV used in the simulation

Symbol	Description	Value	Unit
m	TWV Mass	637	Kg
H	Center of Gravity height to the ground	0.713	m
Tr	Rear Track width	1.154	m
WB	Wheelbase	2.127	m
FWB	Front axle to CoG distance	1.285	m
RWB	Rear axle to CoG distance	0.841	m
rTr	CoG to rear right wheel distance	0.593	m
lTr	CoG to rear left wheel distance	0.560	m

The centre of gravity of the vehicle used in this study is not centered with respect to the X-axis, which is reflected in rear tracks (rTr & lTr) that are not identical (table 3).

The calculations performed on the empty vehicle using both criteria reveal a marginal decrease (-5%) in the SSF with the improved version. This is indicated by the difference between the two rear tracks, which does not exceed 3% of the track width.

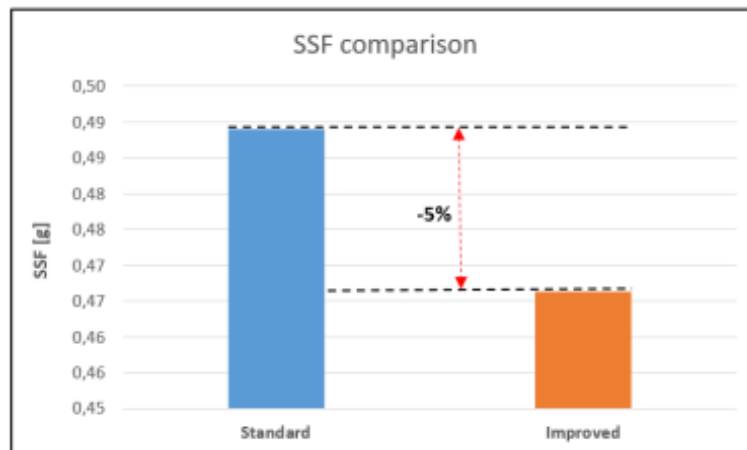


Figure 12. TWV SSF comparison at unladen vehicle

Following our procedure, we will start by loading the vehicle with a specific weight at a predetermined location on the tray (as illustrated in figure 13). This load is unbalanced along the X-axis, involving the sequential loading of the two-stage with 150 kg in the initial stage and an additional 300 kg in the final stage. Subsequently, we will engage in the calculation of the SSF, meticulously exploring both of its versions to ensure a comprehensive evaluation.

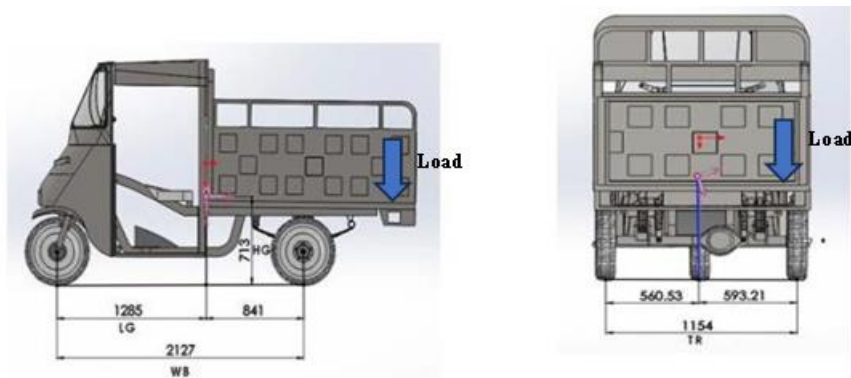


Figure 13. TWV with unsymmetrical Load distribution about X-Axis

An unsymmetrical load along the X-axis within the tray induces an uneven distribution across the two rear wheels of the vehicle. Table 4 provides a visual representation of the percentage distribution of mass on each wheel.

Table 4. Mass distribution on each wheel

Wheel	Mass distribution [%]	
	150 Kg	300 Kg
Front wheel	26	23
Rear right wheel	48	54
Rear left wheel	26	23

The standard version of the SSF does not consider an unsymmetrical load, it focuses on tracking the movement of the center of gravity along the X & Z axis. As the tray is loaded, the center of gravity rises and shifts rearward, this clarifies why the SSF in its standard version hasn't fluctuated significantly across both loading stages (figure 15).

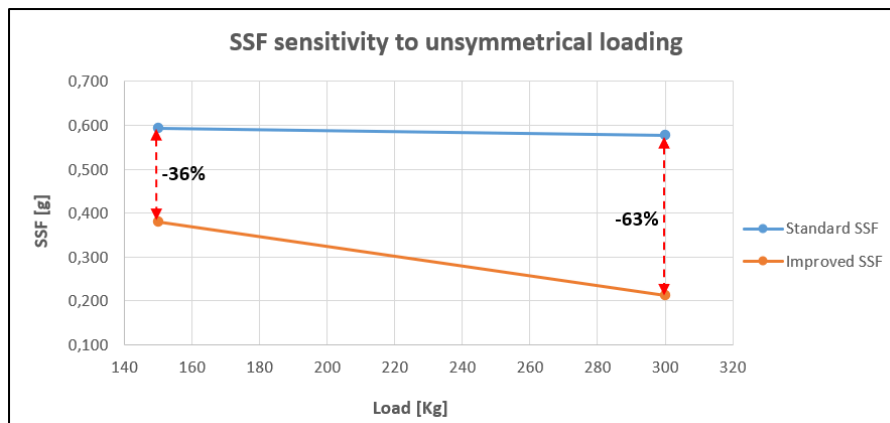


Figure 15. TWV SSF sensitivity to unsymmetrical loading

The improved version of the SSF takes into account the lateral movement of the centre of gravity. Loading the tray in an unsymmetrical manner results in the center of gravity shifting along the Y-axis, leading to a notable degradation in the SSF. Figure 15 illustrates that the degradation of SSF can escalate to considerable levels with the increment of load on the tray.

4- Conclusion

This paper proposes an improvement to the SSF criterion for TWV, aiming to incorporate lateral coordination of the center of gravity. Simulations conducted reveal the vehicle's high sensitivity to unsymmetrical loading, which significantly deteriorates lateral stability. The objective is to raise awareness among engineers and designers of such vehicles about this inherent characteristic and provide them with a means to assess rollover risks during the initial design phases, using only a few architectural parameters.

Enhancing the active safety of terrestrial vehicles is a shared concern among researchers and engineers, leading to various proposals from multiple teams. J. Dižo et al. [19] introduced a new steering system architecture aimed at enhancing the lateral stability of TWV. Their objective was to develop a steering mechanism capable of elevating the lateral stability of TWV to a level comparable to that of Four Wheeled Vehicles. Through analytical demonstrations and simulations, they successfully showcased the promising potential of their concept. R. A. Azim et al. [20] conducted a study focusing on

proposing an active steering system. This system aims to assist drivers in detecting rollover situations and implementing regulations to stabilize the vehicle.

5- Acknowledgement

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